The following is a reproduction of the 1980 Colin Doyle mathematics competition exam paper. It includes some interesting questions, so it would be unfortunate if it were lost to history.

This competition was open to Year 11 students in the Hunter Valley region of New South Wales, Australia. Students below year 11 were not eligible to enter, which seems odd given that most of the questions were accessible well before Year 11. In most cases, if you are below year 11 but understand what the question is asking you to do, you probably have the knowledge required to solve it. Here are some possible exceptions.

- Students sitting this exam in 1980 would have found Q1(c) easy since they had been extensively drilled in the use of logarithms to perform things like multiplication and division without the use of a calculator, probably in Year 9. That no longer happens, so that question may be more challenging to current students.
- Q3(a) concerns arithmetic progressions, and question 4(a) involves some combinatorics. In the 1980 syllabus, these topics were first encountered in Year 11.

The paper distributed to competitors does not include any copyright notice, which I believe means it is in the public domain. I have not had any success in finding any documentation from the era suggesting an author.

The original paper appears to have been produced on a typewriter of dubious mechanical soundness. Some letters were much fainter than their neighbours. Some stems below the baseline vanished. Some symbols and lines were written in by hand. Line spacing was tight, so the whole paper fitted onto one page. It included some typographical errors. It was a challenging read!

Personal computers and word processors have come a long way since 1980, particularly when it comes to formatting mathematics, so I’ve retyped the paper to improve legibility. Notes have been added to correct the typographical errors, which are shown in red. My original attempt introduced some new typographical errors, which have hopefully now been fixed, but let me know if you find any more.

While this is not stated explicitly on the question paper, my memory is that calculators were not allowed. If calculators were allowed, Q1(c) seems pointless.

- Jim Farmer. Nov 2021.

<u>THE NEWCASTLE MATHEMATICAL ASSOC.</u>

<u>THE COLIN DOYLE MEMORIAL PRIZE 1980</u>

All questions may be attempted, but full marks may be obtained without attempting all questions.

Time: 2 hours

1. (a) If $f(x)=x+x^2$,

   (i) evaluate $f(f(2))$, (ii) solve $f(\frac{x-1}{x-2})=6$

   (b) Given $\log_2(3)=1.5851$, find correct to two decimal places

   (i) $\log_2(9)$, (ii) $\log_2(12)$

   (c) S is the set of all integers from 1 to 32 inclusive. In S, A is the set of even multiples of 3, B is the set of odd multiples of 3, C is the set of multiples of 5 and D is the set of multiples of 7.

   Describe the sets

   (i) $A\cap B$ (ii) $(A\cup B)\cap C$ (iii) $(A\cap C)\cup(B\cap D)$

   (d) Find all solution sets (a,b,c,d) for the equations $a^2+bc=1$; $ab+bd=3$; $bc+d^2=4$; $ac+cd=0$.

2. (a) If we multiply together all the numbers from 1 to 100, how many zeros are there on the end of the answer?

   (b) Prove that for integral $m>1, n>1, m^4+4n^4$ is never a prime number.

   (Typist's note: Perhaps this would be clearer as: For integer $m>1, n>1$, prove that m^4+4n^4 is never a prime number.)

3. (a) In an arithmetic progression (A.P.) the first term is 2, the common difference is not zero, the sum of the first *n* terms is S_n and the sum of the next *n* terms is T_n .

   Determine the A.P. if $\frac{T_n}{S_n}$ is independent of *n*.

   (b) A 2-digit number is divided by the sum of its digits. Find the maximum and minimum values of this quotient.

4. (a)

8	6	5	3
7	2		
4			
1			

The numbers 1, 2, 3,…, 8 are to be placed in the cells shown so that each number is greater than the number if any, directly on its right or directly below it. One suitable placement is shown. How many suitable arrangements are there?

(Typist's note: The first sentence is arguably ambiguous. Based on the example pattern given, I think the intended meaning is "each number is greater than the number if any directly on its right AND it is also greater than the number if any directly below it." An alternative interpretation is that where a number has both a number directly to its right and a number directly below it, it only has to be greater than one of those two numbers. However, this alternative interpretation involves an excessively lengthy and tedious listing of cases that would be best solved by a computer rather than by a maths competition student.)

(b) In a certain country the scale of notation contains three extra digits, X, Y, Z interpolated in the ordinary denary scale so that we have the following correspondence.

1	2	3	4	5	6	7	8	9	10	11	12	13
1	2	3	X	4	5	Y	6	7	8	Z	9	10

e.g. our $100=7(13)+9$ is represented by their 77.

(Typist's note: That's what it said! Presumably 77 is a typographical error for Y7.)

(i) Which of our numbers is represented by XYZ?

(ii) What in their notation is the square of their number 1X?

5. ABC is a triangle with $\angle B=2\angle A$. Using the Sine Rule and Cosine Rule prove that

$$b^2=a(a+c)$$

Also prove the same result by producing AB to D so that BD = DC, joining CD and using similar triangles.

(Typist's note: If you perform that construction, you won't find any similar triangles, so it looks like we've got a typographical error. Try changing the part in red so that it reads BD = BC.)